Constrained Bi-objective Surrogate-Assisted Optimization of Problems with Heterogeneous Evaluation Times: Expensive Objectives and Inexpensive Constraints

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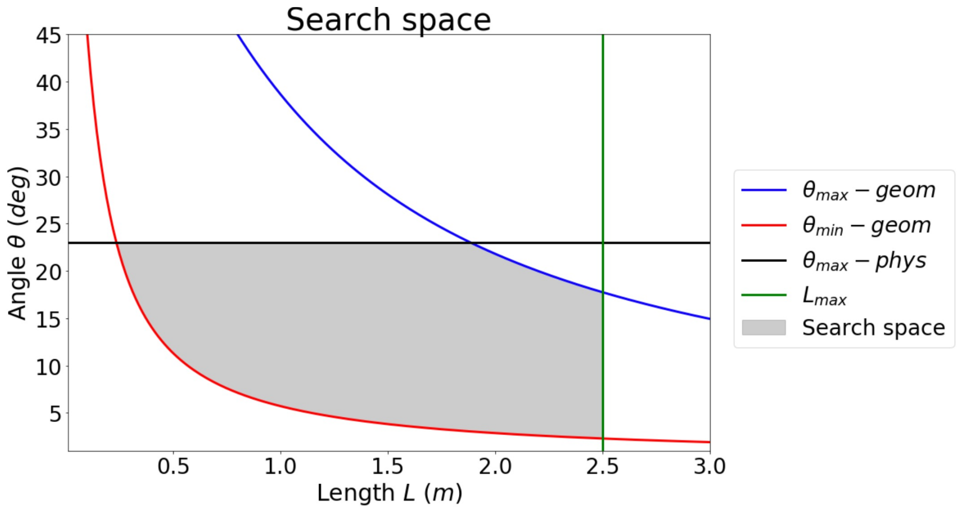
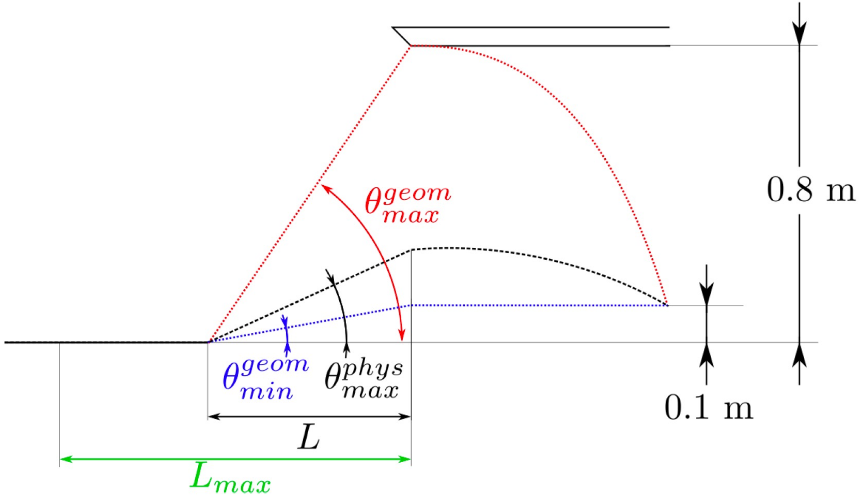
EMO Shenzhen 2021

Motivation: Diffuser Inlet Design Problem



Visualization of Search Space



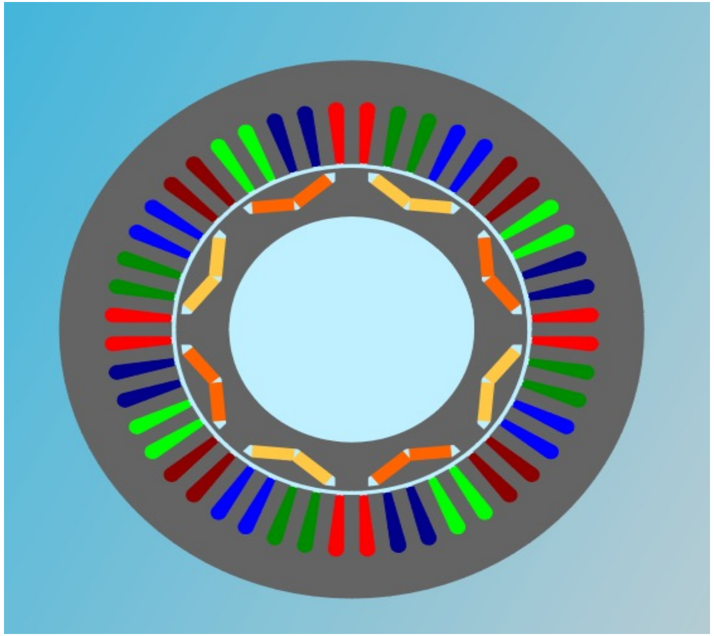
Geometry

https://github.com/jlobatop/GA-CFD-MO

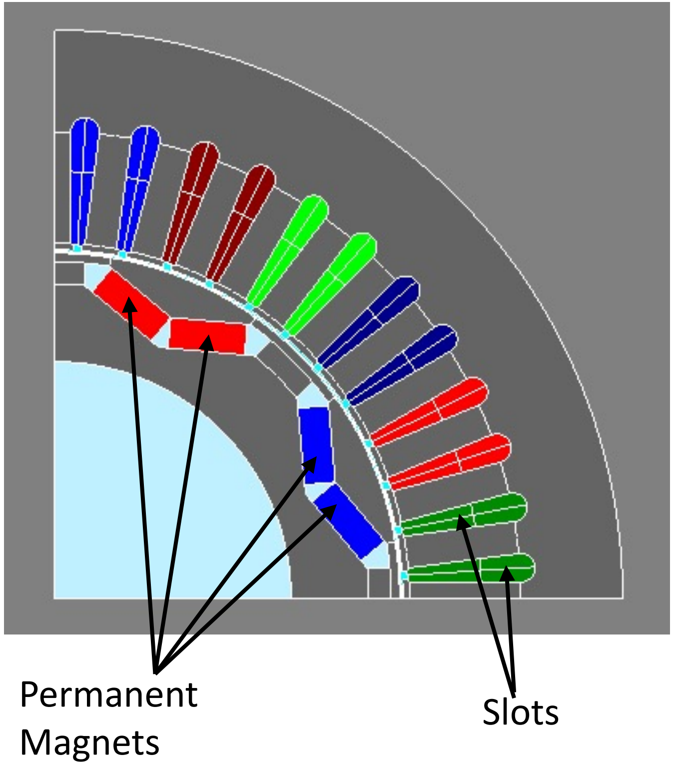
Javier Lobato Perez, Genetic algorithms applied in Computer Fluid Dynamics for multiobjective optimization, PhD Thesis, 2018.

Toyota Prius 2010

Motivation: Electric Machine Design Optimization



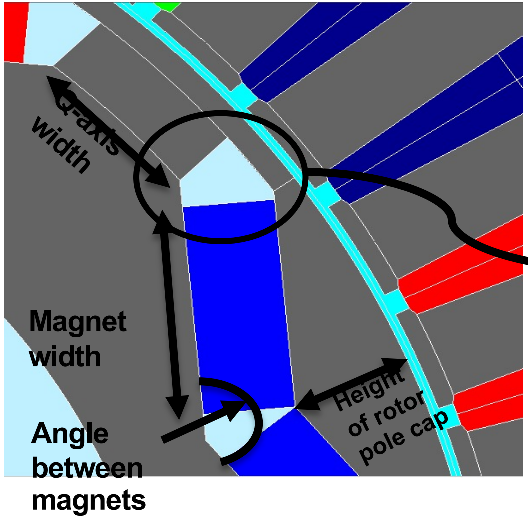
Radial view of the geometry

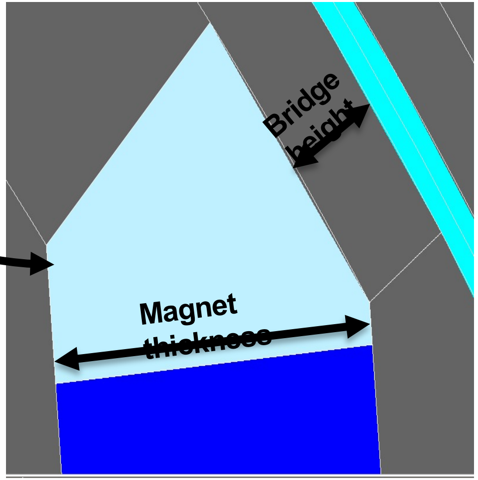


Model from Altair Fluxmotor 2019/Motor Catalog/Automative_Tansport_1/Prius_2010”

Motivation: Electric Machine Design Optimization

- Variables: 10 continuous variables with a precision of 2
- Objectives: (Computationally Expensive)
  - Maximize: Average Torque
  - Minimize: Torque Ripple
- Constraints (Computationally Inexpensive):
  - Satisfying the geometric constraints





Model from Altair Fluxmotor 2019/Motor Catalog/Automative_Tansport_1/Prius_2010”

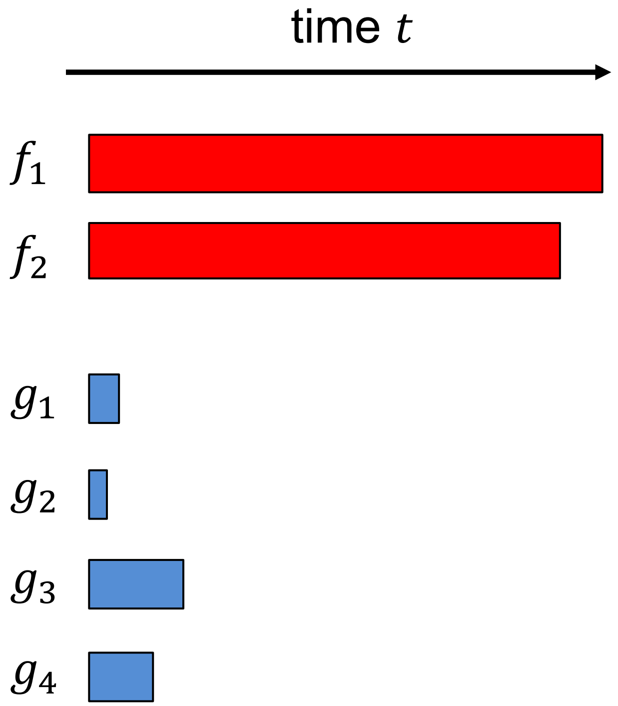
Motivation: Electric Machine Design Optimization

1. $\sqrt{ZIM\_X1^2 + ZIM\_Y1^2} \geq \frac{IM\_ID}{2} + 2$
2. $\sqrt{ZIM\_X4^2 + ZIM\_Y4^2} \leq \frac{IM\_OD}{2} - 1$
3. $ZIM\_X7 - ZIM\_X5 > 0$
4. $ZIM\_Y7 - ZIM\_Y5 > 0$
5. $\sqrt{ZIM\_X7^2 + ZIM\_Y7^2} \leq \frac{IM\_OD}{2} - IM\_T1 - 1$
6. $ZIM\_X8 - ZIM\_X4 \leq 3$
7. $ZIM\_Y8 - ZIM\_Y4 \leq 1$
8. $OS\_WS1 - OS\_WO > 0$
9. $|ZOS\_V1 - ZOS\_VE| \leq ZOS\_VE$

- Simple to calculate
- But challenging to satisfy through manual analysis

Type of Optimization Problems

$$
\begin{array}{llll}
\text{Min/Max} & f_m(x) & m = 1, 2, , \ldots, M; \\
\text{subject to} & g_j \leq 0, & j = 1, 2, \ldots, J; \\
 & h_k = 0, & k = 1, 2, \ldots, K; \\
\multicolumn{2}{l}{x_i^{(L)} \leq x_i \leq x_i^{(U)},} & i = 1, 2, \ldots, N.
\end{array}
$$



Related Work

- Allmendinger, R., Knowles, J.: 'Hang on a minute': Investigations on the effects of delayed objective functions in multiobjective optimization. In: Purshouse, R.C., Fleming, P.J., Fonseca, C.M., Greco, S., Shaw, J. (eds.) Evolutionary multi-criterion optimization. pp. 6–20. Springer Berlin Heidelberg (2013)
- Allmendinger, R., Handl, J., Knowles, J.: Multiobjective optimization: When objectives exhibit non-uniform latencies. European Journal of Operational Research243(2), 497 – 513 (2015)
- Tinkle Chugh, Richard Allmendinger, Vesa Ojalehto, and Kaisa Miettinen. 2018. Surrogate-assisted evolutionary biobjective optimization for objectives with non-uniform latencies. In Proceedings of the Genetic and Evolutionary Computation Conference (GECCO '18). Association for Computing Machinery, New York, NY, USA, 609–616. Thomann, J., Eichfelder, G.: A trust-region algorithm for heterogeneous multiobjective optimization. SIAM Journal on Optimization29(2), 1017–1047 (2019)
- Wang, X., Jin, Y., Schmitt, S., Olhofer, M.: Transfer learning for gaussian process assisted evolutionary bi-objective optimization for objectives with different evaluation times. In: Proceedings of the 2020 genetic and evolutionary computation conference. pp. 587–594. GECCO '20, ACM, New York, NY, USA (2020)

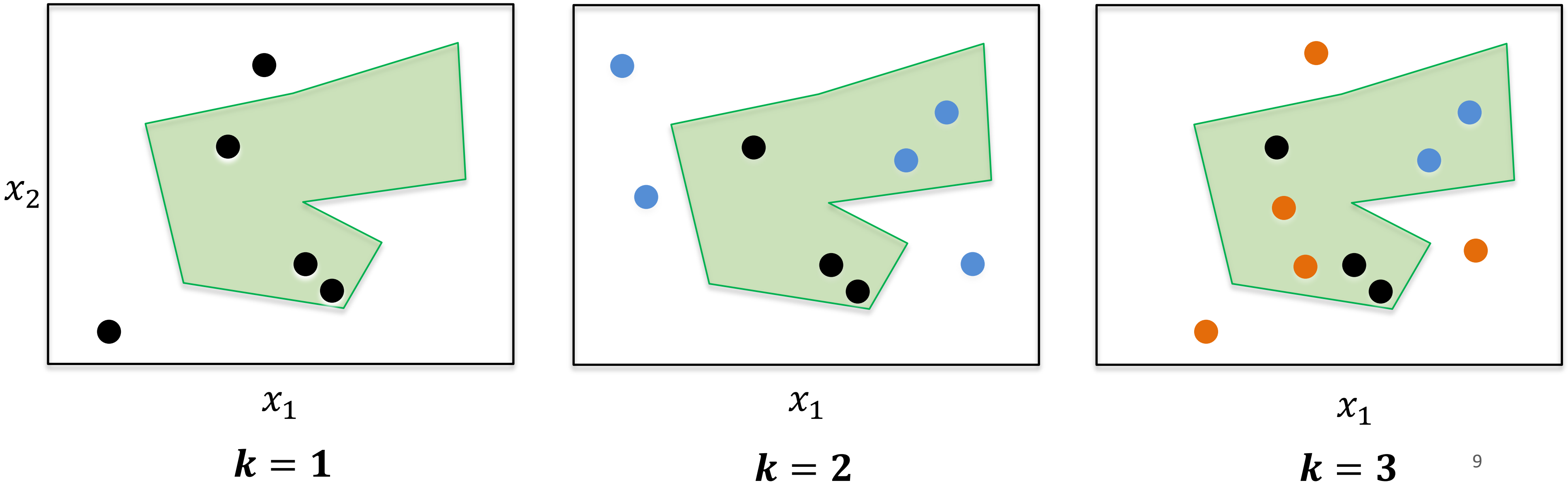
Existing studies focus on unconstrained heterogeneously expensive bi-objective problems

Methodology

- **IC-SA-NSGA-II**
  - **IC:** Inexpensive Constraint(s)
  - **SA:** Surrogate Assisted
  - **NSGA-II:** Baseline Algorithm
- **Initial Design of Experiments:**
  - Rejection Based Sampling (RBS)
  - Niching Genetic Algorithm (NGA)
  - Riesz s-Energy Optimization (Energy)
- **Algorithm Loop:**
  - Exploitation with Surrogate-Bias
  - Exploration through traditional Mating

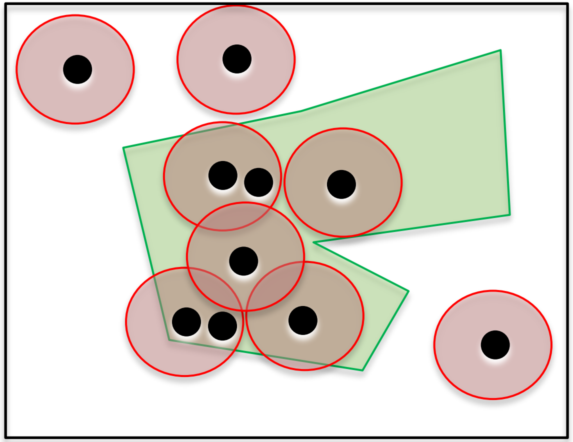
Rejection Based Sampling (RBS)

Use Random or Pseudo Random Sampling and accept a point only if it is feasible.

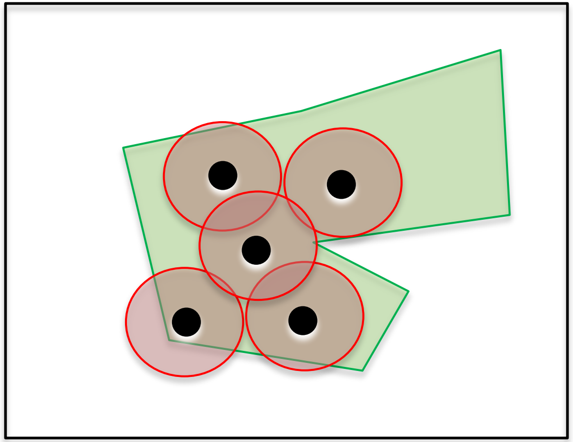


Niching Genetic Algorithm (NGA)

Execute a genetic algorithm with ϵ-clearing where the constraint is the objective.



Environmental Survival

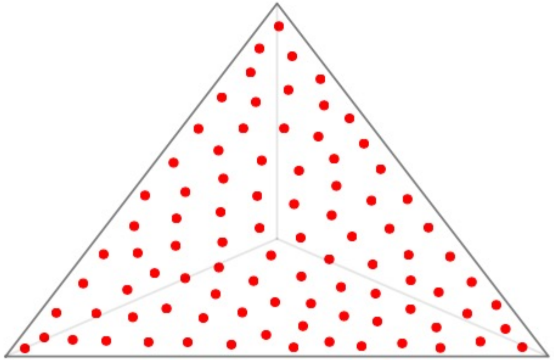


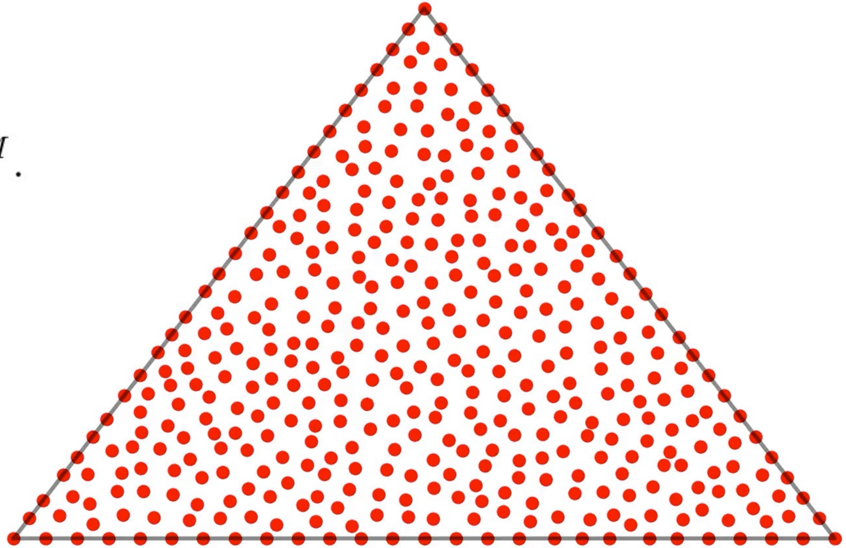
Representative Solutions

Generating Well-Spaced Points on a Unit Simplex for Evolutionary Many-Objective Optimization (TEV, 2020))

$$U(\mathbf{z}^{(i)}, \mathbf{z}^{(j)}) = \frac{1}{\left\|\mathbf{z}^{(i)} - \mathbf{z}^{(j)}\right\|^{s}}.$$

$$U_T(\mathbf{z}) = \frac{1}{2}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n} \frac{1}{\left\|\mathbf{z}^{(i)} - \mathbf{z}^{(j)}\right\|^{s}}, \quad \mathbf{z} \in \mathbb{R}^{n\times M}.$$

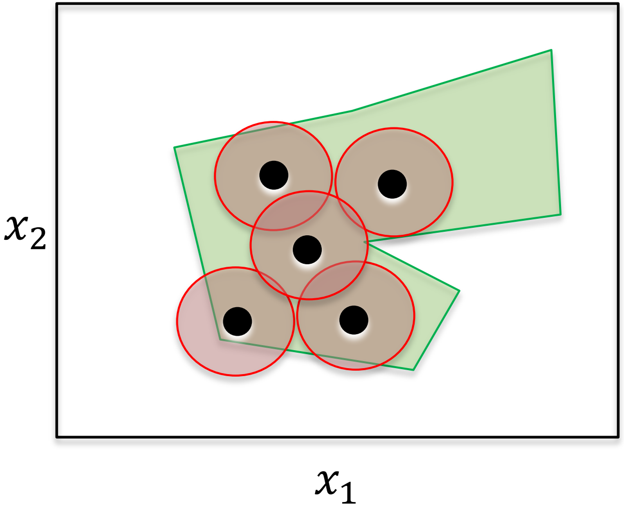




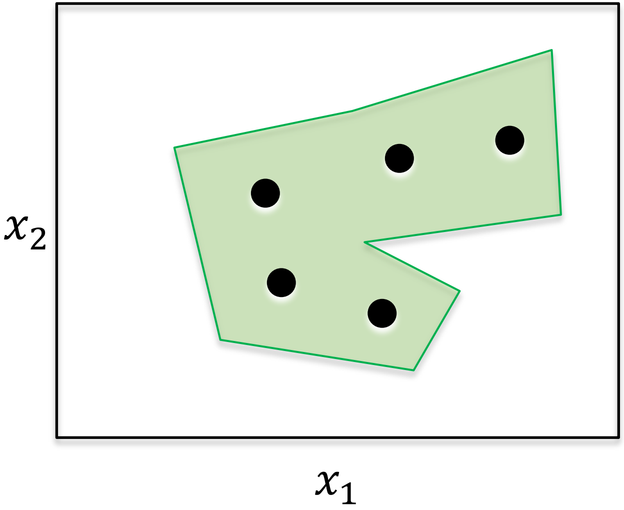
J. Blank, K. Deb, Y. Dhebar, S. Bandaru and H. Seada, "Generating Well-Spaced Points on a Unit Simplex for Evolutionary Many-Objective Optimization," in IEEE Transactions on Evolutionary Computation, doi: 10.1109/TEVC.2020.2992387.

Riesz s-Energy Optimization (Energy)

Improve the result from NGA further by iteratively improving Riesz s-Energy



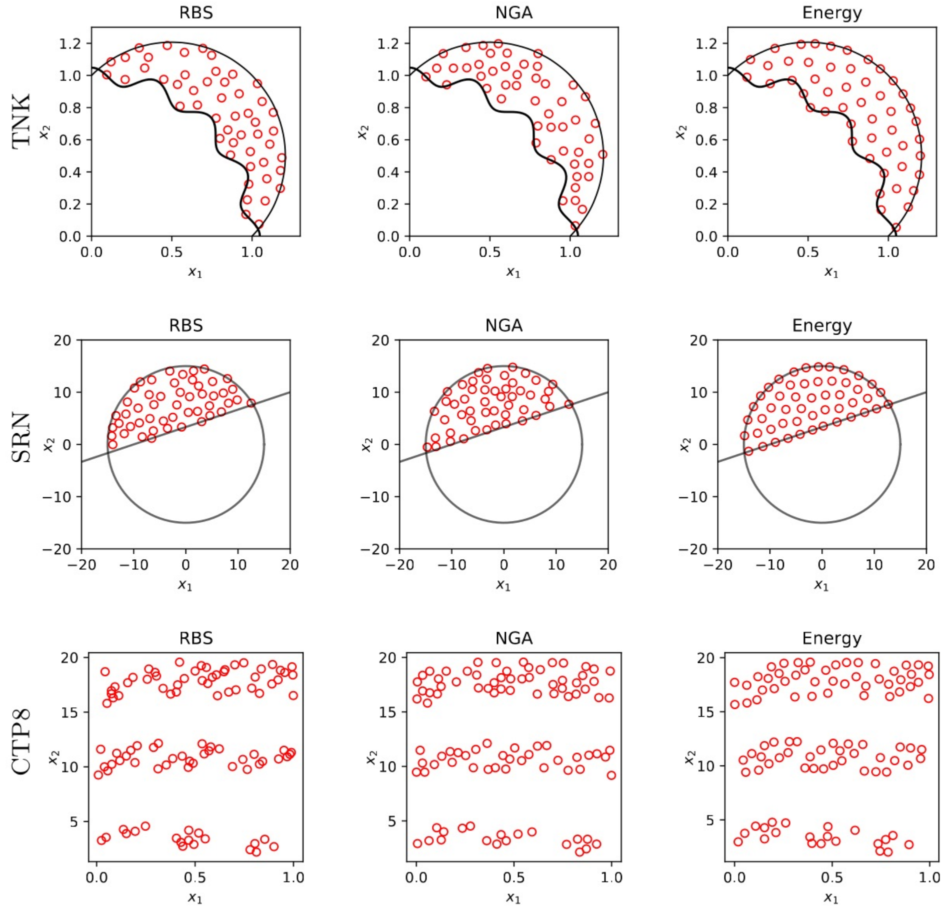
NGA



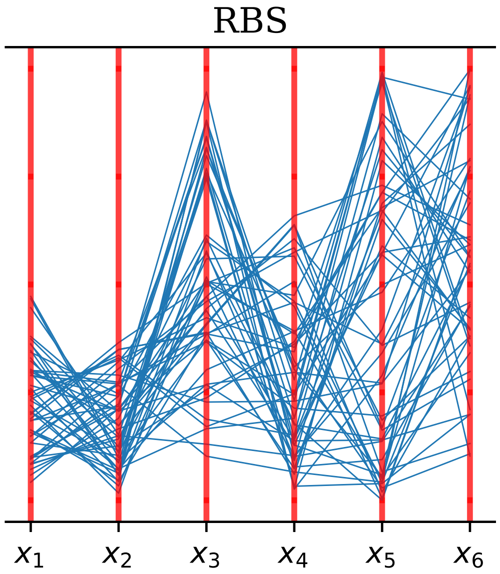
Riesz s-Energy

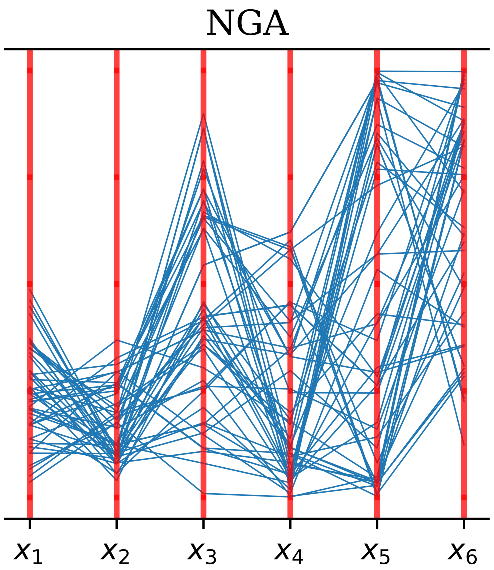
Sampling

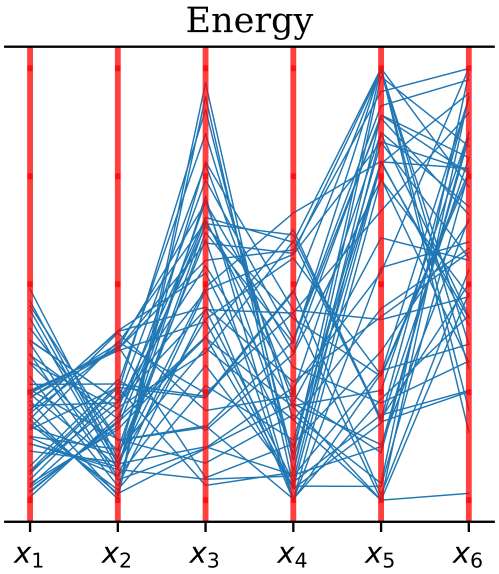
- Results on two-dimensional test problems: TNK, SRN CTP8
- The Riesz s-Energy method achieves a well–spaced point set across all problems
- Different clusters of feasible regions are not an issue (at least for lower dimensional spaces)



What about higher Dimensions?







OSY

Methodology: Pseudo Code

Make use of the fact that the constraints are computationally inexpensive

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Algorithm 1: IC-SA-NSGA-II: **I**nexpensive **C**onstrained **S**urrogate-**A**ssisted NSGA-II.

Input: Number of Variables n, Expensive Objective Function $f(\mathbf{x})$, Inexpensive Constraint Function $g(\mathbf{x})$, Maximum Number of Solution Evaluations $\texttt{SE}^{\max}$, Number of Design of Experiments $N^{\texttt{DOE}}$, Exploration Points $N^{(\texttt{explr})}$, Exploitation Points $N^{(\texttt{exploit})}$, Number of generations for exploitation k, Multiplier of offsprings for exploration s

/* initialize feas. solutions using the inexpensive function g */

$\mathbf{X} \leftarrow$ constrained_sampling('energy', $N^{\texttt{DOE}}, g$)

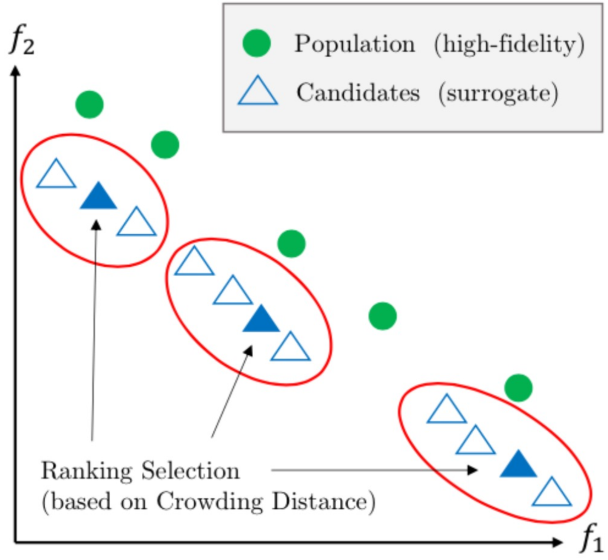
$\mathbf{F} \leftarrow f(\mathbf{X})$

while $|X| < SE^{\max}$ **do**

- /* exploitation using the surrogate */
- $\hat{f} \leftarrow$ fit_surrogate($\mathbf{X}, \mathbf{F}$)
- $(\mathbf{X}^{(\texttt{cand})}, \mathbf{F}^{(\texttt{cand})}) \leftarrow$ optimize('nsga2', $\hat{f}, g, \mathbf{X}, \mathbf{F}, k$)
- $(\mathbf{X}^{(\texttt{cand})}, \mathbf{F}^{(\texttt{cand})}) \leftarrow$ eliminate_duplicates($\mathbf{X}, \mathbf{X}^{(\texttt{cand})}, \mathbf{F}^{(\texttt{cand})}$)
- $C \leftarrow$ cluster('k_means', $N^{(\texttt{exploit})}, \mathbf{F}^{(\texttt{cand})}$)
- $\mathbf{X}^{(\texttt{exploit})} \leftarrow$ ranking_selection($\mathbf{X}^{(\texttt{cand})}, C,$ crowding($\mathbf{F}^{(\texttt{cand})}$))
- /* exploration using mating and least crowded selection */
- $\mathbf{X}', \mathbf{F}' \leftarrow$ survival($\mathbf{X}, \mathbf{F}$)
- $\mathbf{X}^{(\texttt{mat})} \leftarrow$ mating($\mathbf{X}', \mathbf{F}', s \cdot N^{(\texttt{explr})}$)
- $\mathbf{X}^{(\texttt{explr})} \leftarrow$ feas_and_max_distance_selection($\mathbf{X}^{(\texttt{mat})}, \mathbf{X}^{(\texttt{cand})}, X, g$)
- /* evaluate and merge to the archive */
- $\mathbf{F}^{(\texttt{explr})} \leftarrow f(\mathbf{X}^{(\texttt{explr})})$; $\mathbf{F}^{(\texttt{exploit})} \leftarrow f(\mathbf{X}^{(\texttt{exploit})})$;
- $\mathbf{X} \leftarrow \mathbf{X} \cup \mathbf{X}^{(\texttt{explr})} \cup \mathbf{X}^{(\texttt{exploit})}$
- $\mathbf{F} \leftarrow \mathbf{F} \cup \mathbf{F}^{(\texttt{explr})} \cup \mathbf{F}^{(\texttt{exploit})}$

end

Methodology: Exploitation



(a) Exploitation: Select solutions from the candidates set obtained by optimizing on the surrogate.

while $|X| < SE^{\max}$ **do**

  /* exploitation using the surrogate */

  $\hat{f} \leftarrow \texttt{fit\_surrogate}(\mathbf{X}, \mathbf{F})$

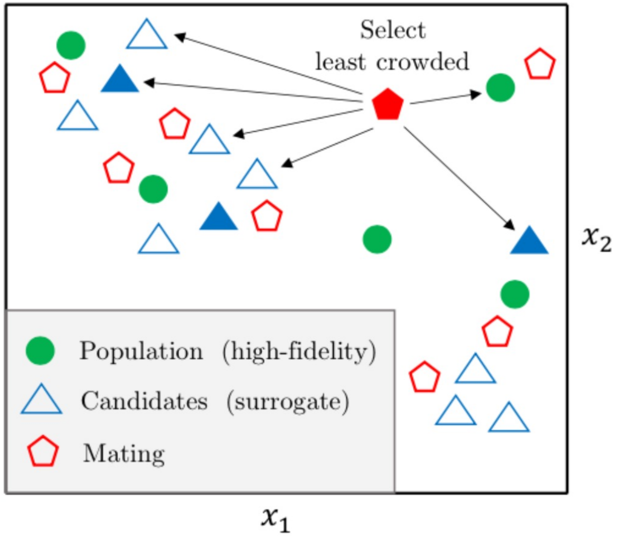
  $(\mathbf{X}^{(\text{cand})}, \mathbf{F}^{(\text{cand})}) \leftarrow \texttt{optimize}(\text{'nsga2'}, \hat{f}, g, \mathbf{X}, \mathbf{F}, k)$

  $(\mathbf{X}^{(\text{cand})}, \mathbf{F}^{(\text{cand})}) \leftarrow \texttt{eliminate\_duplicates}(\mathbf{X}, \mathbf{X}^{(\text{cand})}, \mathbf{F}^{(\text{cand})})$

  $C \leftarrow \texttt{cluster}(\text{'k\_means'}, N^{(\text{exploit})}, \mathbf{F}^{(\text{cand})})$

  $\mathbf{X}^{(\text{exploit})} \leftarrow \texttt{ranking\_selection}(\mathbf{X}^{(\text{cand})}, C, \texttt{crowding}(\mathbf{F}^{(\text{cand})}))$

Methodology: Exploration



(b) Exploration: Select from a solution set obtained through evolutionary operators by maximizing the distance to existing solutions and candidates.

/* exploration using mating and least crowded selection */

$\mathbf{X}', \mathbf{F}' \leftarrow \texttt{survival}(\mathbf{X}, \mathbf{F})$

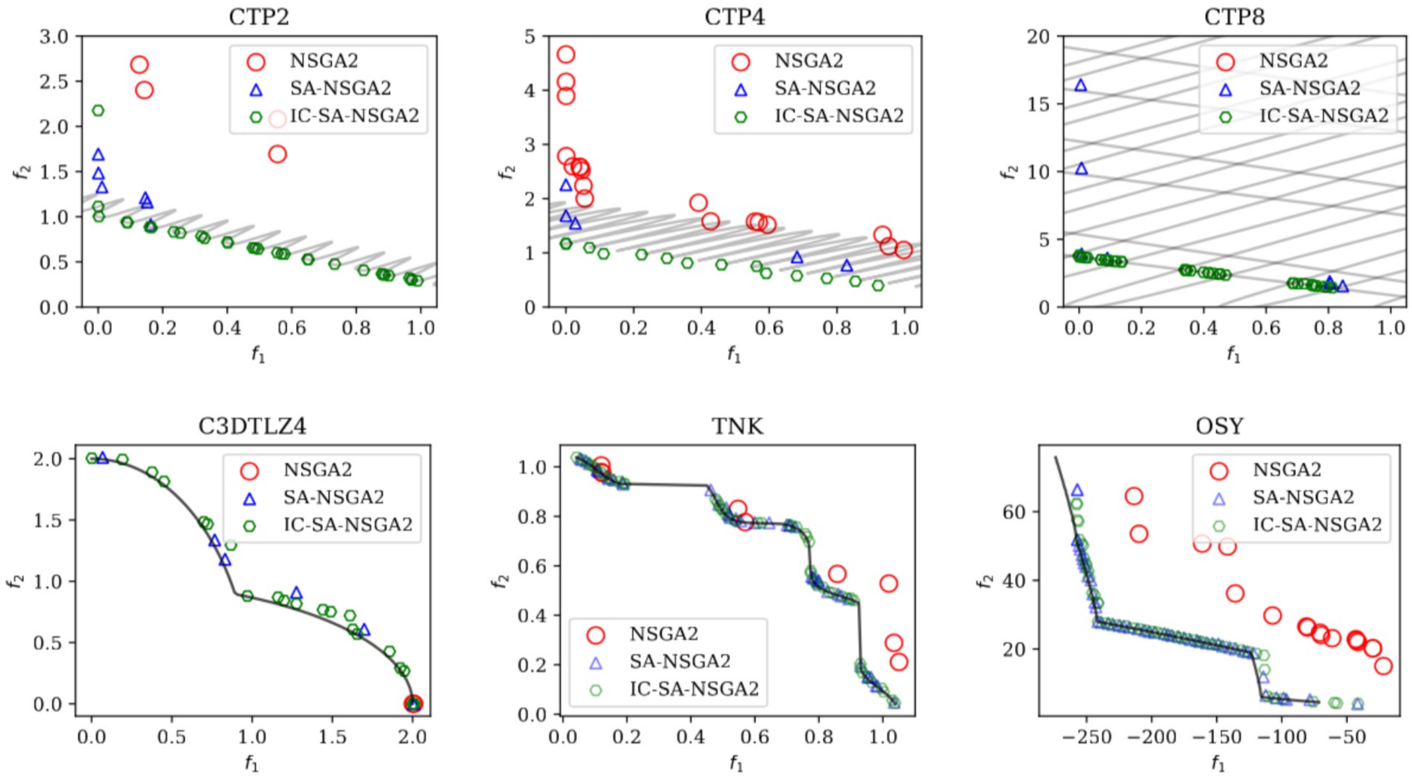
$\mathbf{X}^{(\texttt{mat})} \leftarrow \texttt{mating}(\mathbf{X}', \mathbf{F}', s \cdot N^{(\texttt{explr})})$

$\mathbf{X}^{(\texttt{explr})} \leftarrow \texttt{feas\_and\_max\_distance\_selection}(\mathbf{X}^{(\texttt{mat})}, \mathbf{X}^{(\texttt{cand})}, X, g)$

Results: Constrained Bi-objective Optimization Problems

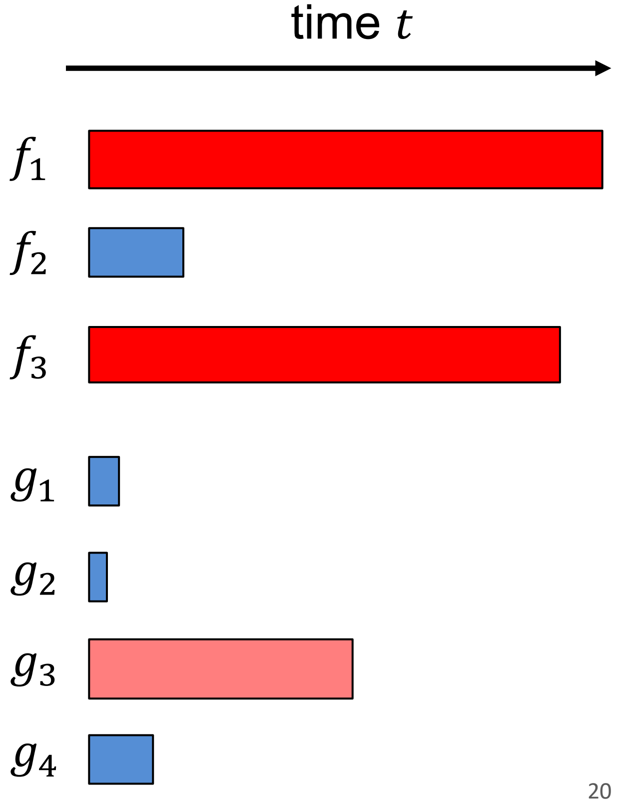
Problem	Variables	Constraints	SE^{max}	NSGA-II	SA-NSGA-II	IC-SA-NSGA-II
CTP1	10	2	200	3.6399	**0.0237**	**0.0196**
CTP2	10	1	200	1.4422	0.1721	**0.0173**
CTP3	10	1	200	1.2282	0.2752	**0.0357**
CTP4	10	1	400	0.8489	0.3969	**0.0736**
CTP5	10	1	400	0.7662	0.1145	**0.0139**
CTP6	10	1	400	7.7155	0.1909	**0.0117**
CTP7	10	1	400	1.5517	0.0164	**0.0032**
CTP8	10	2	400	11.6452	0.5963	**0.0074**
OSY	6	6	500	0.4539	**0.0273**	**0.0381**
SRN	2	2	200	0.0263	**0.0112**	**0.0108**
TNK	2	2	200	0.1281	0.0200	**0.0092**
C2DTLZ2	12	1	200	0.3787	0.1185	**0.0484**
C3DTLZ4	7	2	200	0.2622	0.1210	**0.0481**
CAR	7	10	200	0.2362	0.0168	**0.0147**

Results: Constrained Bi-objective Optimization Problems



Future Work: Heterogeneously Expensive Objectives/Constraints

$$
\begin{array}{llr}
\text{Min/Max} & f_m(x) & m = 1, 2, , \ldots, M; \\
\text{subject to} & g_j \leq 0, & j = 1, 2, \ldots, J; \\
 & h_k = 0, & k = 1, 2, \ldots, K; \\
x_i^{(L)} \leq x_i \leq x_i^{(U)}, & & i = 1, 2, \ldots, N.
\end{array}
$$



Conclusions

- Efficiently Handling Inexpensive Constraints makes sense and can significantly improve the performance of an algorithm
- The Riesz s-Energy concept is an effective concept for creating a feasible space-filling set of points
- Expensive objectives and inexpensive constraints and It is a special case of computationally expensive optimization problems and concepts dealing with with varying expensiveness need to be investigated

Questions?